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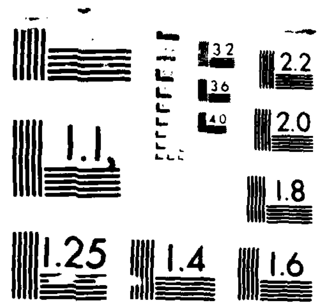

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Technical Memorandum

ENVELOPE DETECTION OF DISCRETE-TIME BAND-LIMITED SIGNALS USING ANALYTIC SIGNAL REPRESENTATIONS

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Prepared by: J. Matthew Tattersall
J. Matthew Tattersall
Surface Ship Sonar
Department

ABSTRACT

This memorandum discusses basic concepts relating to analytic signal generation and envelope detection by discrete-time methods.

ADMINISTRATIVE INFORMATION

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I. INTRODUCTION

Often when one is analyzing band-pass data, it may be desirable to look at only the envelope of the signal. This is certainly the case for communication systems using pulse amplitude modulation, where all the necessary information is contained in the envelope of the signal. For data such as crosscorrelations or autocorrelations, often much can be learned from only the envelope of the resulting correlation function.

This short note will discuss the calculation of the envelope of a real band-pass signal using an analytic signal representation of the signal. Analytic signal representations are common in the development of radar [1], sonar [2], and more recently, geophysical signal processing techniques [3],[4].

II. ANALYTIC AND PHYSICAL SIGNALS

The analytic signal representation of a real signal is a complex signal. As with any complex signal, there is a magnitude only envelope that is modulated by a complex exponential. However, as will be discussed subsequently, there is a special relationship between the real and imaginary parts of an analytic signal that separates analytic signals from the general class of complex signals.

Given a real, band-limited, discrete-time signal, $x(n)$, the analytic signal representation of $x(n)$ is defined as:

$$\tilde{x}(n) = x(n) + j\hat{x}(n) \qquad (1),$$

where $\hat{x}(n)$ is the Hilbert Transform of the sequence $x(n)$. This is the relationship that distinguishes analytic signals from signals that are simply complex.

In polar notation, Eq.(1) can be expressed as:

$$\tilde{x}(n) = a(n)\, e^{j\theta(n)} \qquad (2).$$

From (1) and (2), it is apparent that:

$$a(n) = \left[x(n)^2 + \hat{x}(n)^2 \right]^{1/2} \qquad (3),$$

and:

$$\theta(n) = \arctan(\hat{x}(n)/x(n)) \qquad (4).$$

The physical signal (i.e., the real positive valued envelope) is given by Eq.(3). The phase of the analytic signal is given by Eq. (4).

The phase can be rewritten as:

$$\theta(n) = \psi(n) + n\omega_c \quad (4a)$$

where ω_c is the "carrier" or center frequency of the band-limited signal. (Note that frequency variable is an angular frequency, $\omega = 2\pi f/f_s$, where f_s is the sampling frequency.) Using Eq. (4a), the analytic signal can be expressed as:

$$x(n) = [\ a(n)e^{j\psi(n)}\ ]e^{jn\omega_c} \quad (1a).$$

The quantity within brackets of Eq. (1a) is usually referred to as the low-pass complex envelope of the real band-limited signal. It is easily shown that the real and imaginary parts of the low-pass complex envelope are the inphase and quadrature components of the quadrature demodulation of x(n). The important point is that whether the physical signal is calculated from the analytic signal or from the quadrature demodulated signal the results are equivalent. In order for the phases to be equal the analytic signal must be demodulated by $e^{-jn\omega_c}$.

III. DISCRETE HILBERT TRANSFORMATIONS

Because of the special relationship between a signal and its Hilbert Transform found in the analytic representation of the signal, some properties of the discrete Hilbert Transform will be discussed.

For the purpose of this discussion, the discrete Hilbert Transform of the sequence x(n) will be considered to be the output of a linear discrete time system with x(n) as the input. This implies that the Hilbert Transform of a signal can be obtained by convolving the signal with a particular filter impulse response:

$$\hat{x}(n) = x(n) * h(n) \quad (5),$$

where * denotes discrete-time convolution and h(n) is the pulse response function.

Equivalently, there is a frequency domain representation of the system. The frequency domain transfer function that char-

acterizes this system is shown in Fig. 1 and is expressed as [5]:

$$H(e^{j\omega}) = \begin{cases} -j & 0 \leq \omega < \pi \\ j & -\pi \leq \omega < 0 \end{cases} \qquad (6).$$

As can be seen from Eq. (6), the transfer function operates as a 90° phase shifter (ie., $j = e^{j\pi/2}$).

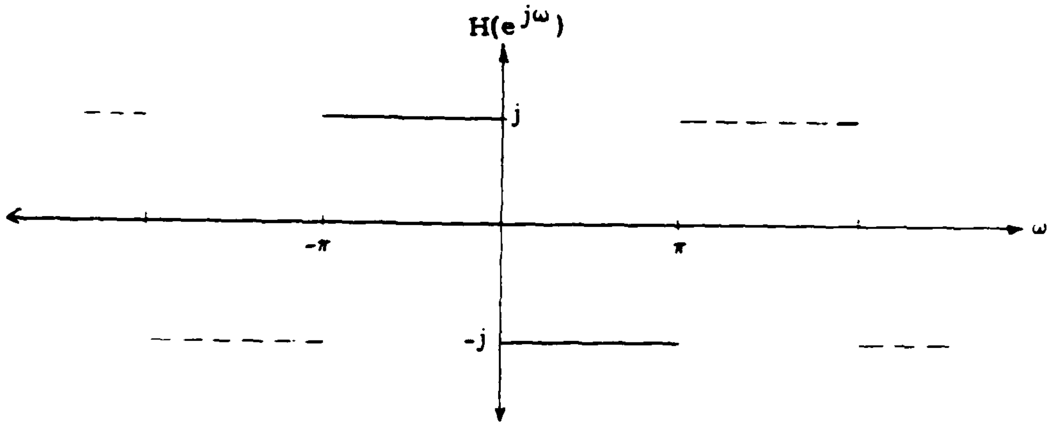

Figure 1. Transfer function for discrete Hilbert Transform.

The unit pulse response corresponding to the above transfer function can be shown to be:

$$h(n) = \frac{\sin^2(n\pi/2)}{(n\pi/2)} \qquad (7a),$$

or equivalently:

$$h(n) = \begin{cases} 2/n\pi & n \text{ odd} \\ 0 & \text{all other } n \end{cases} \qquad (7b).$$

The Hilbert Transform pulse response (Eq. (7)) is shown in Figure 2.

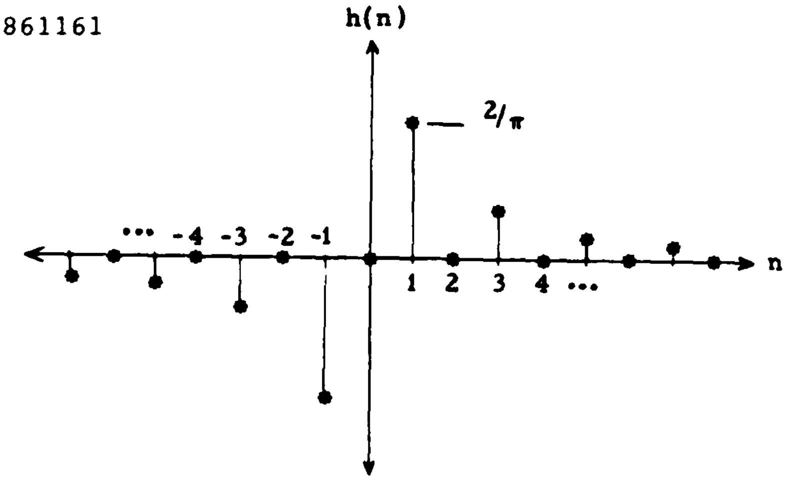

Figure 2. Hilbert Transform impulse response.

As can be seen from Eq. (7), the Hilbert Transform system pulse response is noncausal and belongs to a class of responses known as infinite impulse response (IIR) filters. Because the response is not realizable and because of ripple caused by the discontinuities in Eq. (6), (i.e., the Fourier Transform of (7) approaches (6) only in a least squares sense because of Gibbs' phenomena), the ideal discrete-time Hilbert Transform is similar to the ideal low-pass filter; more useful for theoretical considerations than as a computational tool. Realizable filters which closely approximate the ideal Hilbert Transform (without the undesirable ripple) are easily designed and implemented [5],[6],[7].

IV. LINEAR SYSTEM REPRESENTATION OF ANALYTIC SIGNALS

Using Eq. (1) and the linear system representation of the Hilbert Transform (Eq. (5)), an analytic signal can be considered to be the output of the linear discrete-time system shown in Figure 3. The pulse response of this system can be obtained directly from the system representation (Fig. 3) by noting that:

$$g(n) = \delta(n) + jh(n) \qquad (8a),$$

$$= \begin{cases} 1 & n = 0 \\ 2j/n\pi & n \text{ odd} \\ 0 & \text{all other } n \end{cases} \qquad (8b).$$

Again as was the case for the discrete Hilbert Transform, the desired filter response is not realizable. Approximations to the desired response may be implemented by techniques such as pulse response truncation or windowing and frequency sampling.

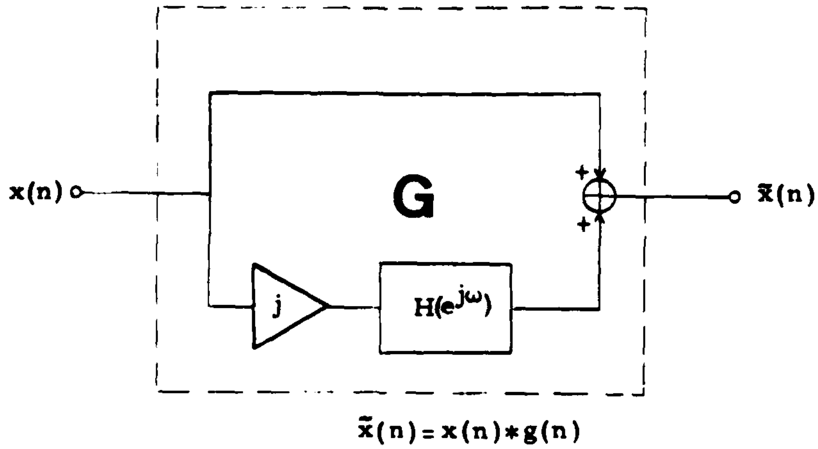

Figure 3. Linear discrete-time system to generate an analytic signal.

The frequency domain representation of the filter which generates the analytical signal is:

$$G(e^{j\omega}) = \begin{cases} 2 & 0 \leq \omega < \pi \\ 0 & -\pi \leq \omega < 0 \end{cases} \tag{9}$$

It is in the frequency domain (see Fig. 4) that we see why the analytic signal representation of a real signal is so convenient. The analytic signal does not contain any of the "negative" frequency components of the original signal and has simply doubled the "positive" frequency components. This property is very useful in reducing the sampling required for band-pass signals [2],[6],[7].

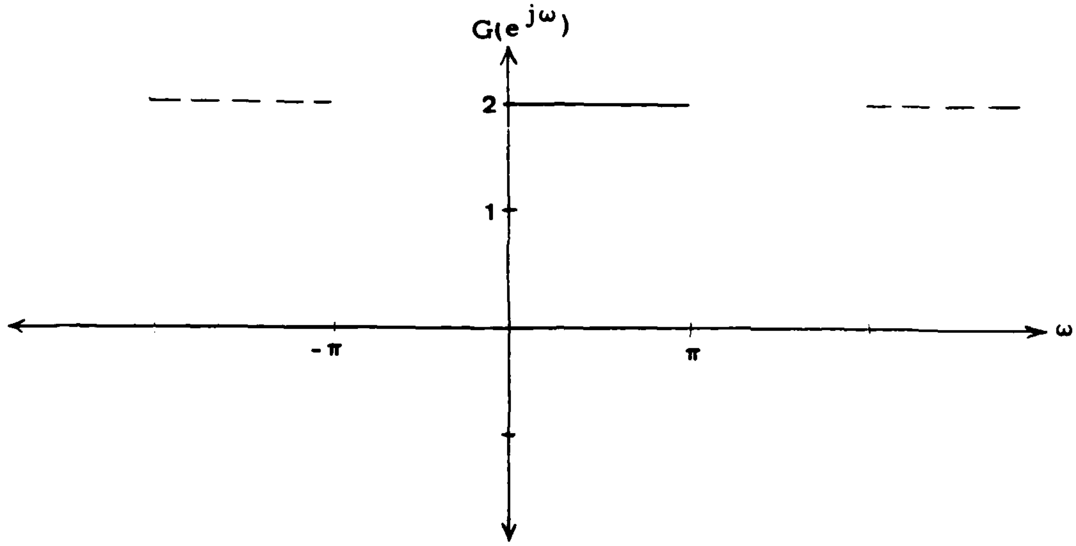

Figure 4. Transfer function for the linear system which generates an analytic signal (see Fig. 3).

V. IMPLEMENTATION BY FREQUENCY SAMPLING

In order to actually implement the linear discrete time system which generates an analytic signal, the transfer function:

$$G(e^{j\omega}) = \begin{cases} 2 & 0 \leq \omega < \pi \\ 0 & -\pi \leq \omega < 0 \end{cases} \qquad (9)$$

must be approximated with a realizable filter. One general technique for this type of filter design is known as "frequency sampling" [5],[6]. Essentially, the desired transfer function is sampled at N equi-spaced points on the unit circle and then the Discrete Fourier Transform (DFT) is used to generate an N point finite impulse response (FIR) filter.

For the case considered here, the sampling is particularly easy, giving:

$$\tilde{G}(k) = \begin{cases} 2 & k=0,1,2,\ \dots\ ,N/2 - 1 \\ 0 & k=N/2,\ \dots,\ N-1 \end{cases} \qquad (10).$$

From this sampled version of the transfer function, it can be shown that the resulting complex N point inverse DFT is:

$$\tilde{g}(n) = \begin{cases} 1 & n=0 \\ 2(1+j\cot(n\pi/N))/N & 0< n \text{ odd} \leq N-1 \\ 0 & 0< n \text{ even} \leq N-1 \end{cases} \qquad (11).$$

The magnitude of Eq. (11) is shown in Figure 5. Note the circularity that is implicit in the use of the DFT.

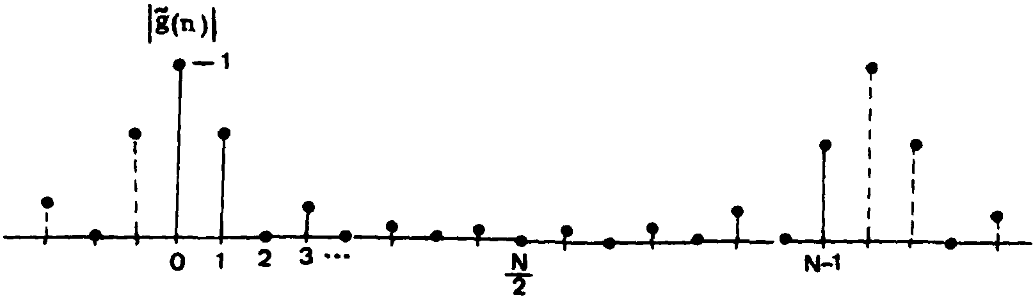

Figure 5. FIR pulse response for frequency sampling filter (note periodicity outside 0 to N-1).

The sequece $\tilde{g}(n)$ is an N point approximation to the infinitely long sequence $g(n)$ (i.e., the exact inverse transform of $G(e^{j\omega})$). Because of the sampling done in the frequency domain, $\tilde{g}(n)$ is a time aliased version of $g(n)$:

$$\tilde{g}(n) = \sum_{m=-\infty}^{\infty} g(n + mN) \qquad (12)$$

It is interesting to note from Eq. (11) that, since for small values of x, $\tan(x) \approx x$, then:

$$\lim N \to \infty \text{ of } \frac{2(1 + j\cot(n\pi/N))}{N} = \frac{2j}{n\pi} \qquad (13).$$

Both Eqs. (12) and (13) show that $\tilde{g}(n)$ is a good approximation of $g(n)$ for large N.

Basically there are two ways that an FIR filter can be implemented numerically; in the time domain as a linear discrete-time convolution or in the frequency domain (via the FFT) by multiplication. To properly use Eq. (11) in a time domain convolution, $\tilde{g}(n)$ must first be circularly rotated by (N-1)/2 points so that the peak of the pulse response is at it's center. This can be thought of as the delay necessary to realize the filter $g(n)$.

In the frequency domain, it is tempting to use $\tilde{G}(k)$ directly and simply "zero out" the DFT/FFT components of $X(k)$ for indices $N/2 \leq k \leq N-1$. However, this actually results in a circular discrete-time convolution of $\tilde{g}(n)$ with $x(n)$, not the desired linear convolution. The wraparound problem can be overcome by zero padding the input signal and using a larger FFT size.

Even though $G(e^{j\omega})$ is the exact system to generate the analytic signal, it may be desirable to implement a slightly different system. This is due to the discontinuities in the transfer function $G(e^{j\omega})$ at $\omega = 0$ and $\omega = \pi$ which will cause ripple in any realization of the system. In fact as is the case for the Hilbert Transform and the ideal low pass filter, the inverse Fourier Transform of the "exact filter" $g(n)$, only converges to $G(e^{j\omega})$ in a least squares sense. For an approximation such as $\tilde{g}(n)$, there is always undershoot and overshoot at the discontinuities. This ripple can be forced to occupy an arbitrarily small band by increasing N, however this is a large computational price to pay. A much better solution to the problem is to eliminate the discontinuties in $G(e^{j\omega})$ by using transition bands [5],[6]. Actually as a practical consideration, ripple may not be as troublesome of a problem as indicated above if the energy of a band-limited signal is centered about $\omega = \pi/2$ and there is little energy near the band edges ($\omega = 0$ and $\omega = \pi$) where the ripple is most severe.

Finally, it should be pointed out that there are many other techniques that could be used to design and implement this particular nonrealizable system. Most of these techniques use efficient ways to calculate the Hilbert Transform required [7].

VI. SUMMARY

The connection between the analytic signal, the discrete Hilbert Transform, quadrature demodulation and the physical signal for band-limited discrete-time signals has been reviewed. For the purposes of this work the analytic signal has been considered to be the output of a nonrealizable linear system. Some details of a frequency sampling filter approximation of this system have been discussed.

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J. Matthew Tattersall
Environmental Acoustic Research &
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